Project Maths Geometry Notes

The areas that you need to study are:

(i) Geometry Terms: Axiom, theorem, proof , corollary, converse, implies

(ii) Theorems: The exam has two questions on geometry which can choose from:

a. Using the old Junior Cert Ordinary level theorems, (which is a difficult question)
b. Using the Project maths theorems

(iii) Constructions: Old Junior Cert Ordinary Level Constructions.
Project Maths Leaving Cert Ordinary Level Constructions.
(See Geometry Notes 2 for constructions.)

(iv) Enlargements: (See Geometry Notes 3 for constructions.)

(i) Terms

N.B. You must learn these definitions and be able to reproduce them in the exam.

Axiom: An **axiom** is a statement which we can accept without any proof. Knowing axioms is essential to proving theorems in geometry. An example of an axiom is that a straight line has 180^o.

Theorem: A **theorem** is a rule that has been proved by following a certain number of logical steps or by using a previous theorem or axiom that you already know. At ordinary level you will not have to prove theorems, instead you will use them to solve problems.

Proof: A **proof** is a series of logical steps which we use to prove a theorem.

Corollary: A **corollary** is a statement that follows from a theorem. We study corollary 6 which follows on from Theorem 20.

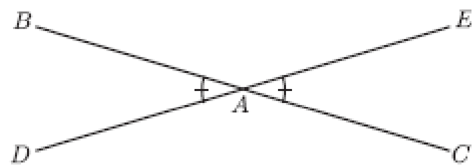
Implies: Implies is a term we use in a proof when we can write down a fact which has been proven from our previous statements. The symbol form implies is $\Rightarrow$.

Congruent: Two triangles are congruent if they are identical in size and shape. The proofs of congruency are **SSS, ASA, SAS and RHS**. (see text book page 304).

Converse: The converse of a theorem is made by switching the statement around.
For example in Theorem 2 (Isosceles Triangles):
Theorem: In an isosceles triangle the angles opposite the equal sides are equal.
Converse: If two angles are equal, then the triangle is isosceles.
N.B. the converse of a statement may not be true. For example two congruent triangles have the same area, however two triangles can have the same area but may not be congruent. This was asked in a past Project Maths exam question.

(ii) (a) Old Junior Cert Ordinary Level Theorems

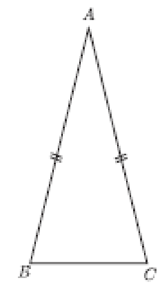
Vertically Opposite Angles
Vertically opposite angles are equal in measure.



∠BAD = ∠EAC
∠BAE = ∠CAD

Isosceles Triangles
(1) In an isosceles triangle the angles opposite the equal sides are equal.
(2) Conversely, If two angles are equal, then the triangle is isosceles.

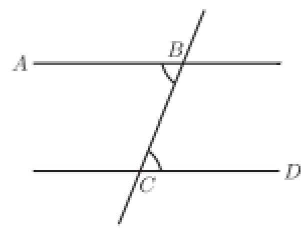
∠ABC = ∠ACB



Alternate Angles (Z angles).
Suppose that A and D are on opposite sides of the line BC:
if AB||CD, then ∠ABC = ∠BCD.

Or conversely

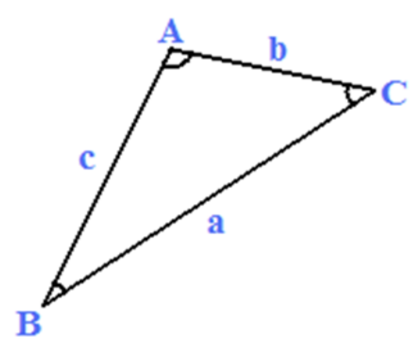
If ∠ABC = ∠BCD, then AB||CD



Angles marked are equal

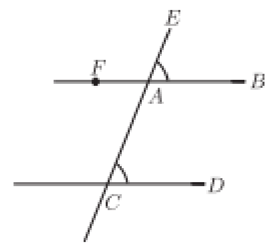
Angles in a triangle Sum to 180^{o}
The angles in any triangle add up to 180°

∠A + ∠B + ∠C = 180°



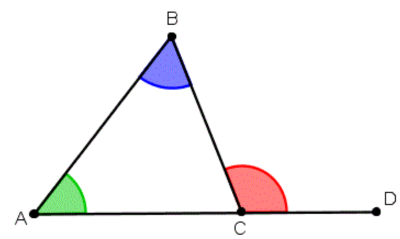
Corresponding Angles (Slide angles or F angles)
Two lines are parallel if corresponding angles are equal.

$$\angle EAB = \angle ACD$$



Exterior Angle
Each exterior angle of a triangle is equal to the sum of the interior opposite angles.

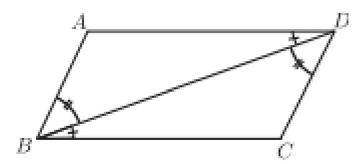
$$\angle BCD = \angle BAC + \angle ABC$$



Parallelogram
In a parallelogram, opposite sides are equal, and opposite angles are equal.

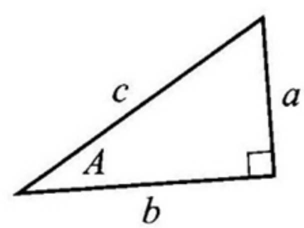
$$|\angle ABC| = |\angle ADC| \;\;\&\;\; |\angle BAD| = |\angle BCD|$$

$$|AB| = |CD| \quad \& \quad |AD| = |BC|$$



Theorem of Pythagoras
In a right-angle triangle the square of the hypotenuse is the sum of the squares of the other two sides.

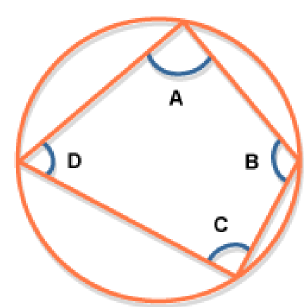
$$c^2 = a^2 + b^2$$



Cyclic Quadrilateral.
If ABCD is a cyclic quadrilateral, then opposite angles sum to 180◦.

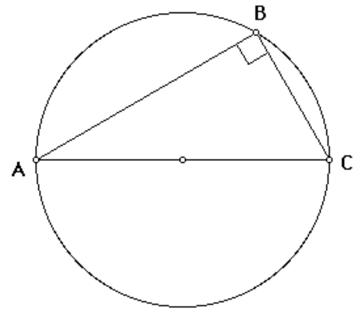
$$\angle D + \angle B = 180^o$$

$$\angle A + \angle C = 180^o$$



Angle in a semicircle is a right angle.

If AC is a diameter of a circle, and B is any other point of the circle, then
$\angle ABC = 90^\circ$



AC a diameter, then Angle B = 90 degrees

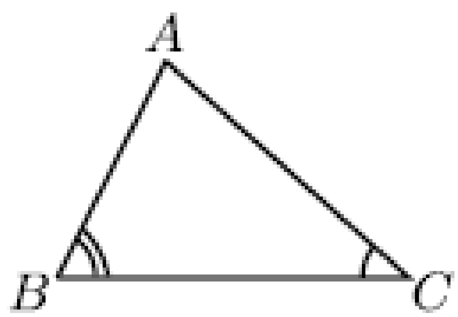
(ii) (b) Project Maths Theorems

You are required to know the following theories and corollaries and be able to apply them in answering questions.

Theorem 7. (Textbook Page 313)
(1) In ΔABC, suppose that side $|AC| > $ side $|AB|$. Then $|\angle ABC| > |\angle ACB|$. In other words, the angle opposite the greater of two sides is greater than the angle opposite the lesser side.

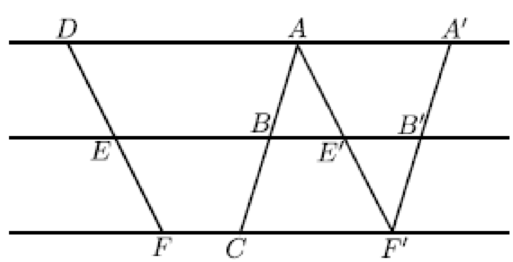
The largest angle is opposite the largest side and the smallest angle is opposite the smallest side.

(2) Conversely, if $|\angle ABC| > |\angle ACB|$, then $|AC| > |AB|$. In other words, the side opposite the greater of two angles is greater than the side opposite the lesser angle.

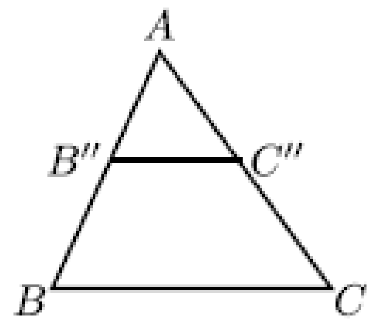


Theorem 8. (Triangle Inequality). **(Textbook Page 314)**
Two sides of a triangle are together greater than the third.
This theorem implies that one side of a triangle must always be smaller than the other two sides added together.

Theorem 11. (Textbook Page 314)
If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.
(A transversal is a line that intersects (crosses over) two or more other lines)
For example if three parallels lines divide a transversal into two equal parts they must do the same for any other transversal.



Theorem 12.
Let ABC be a triangle. If a line is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts AC in the same ratio.



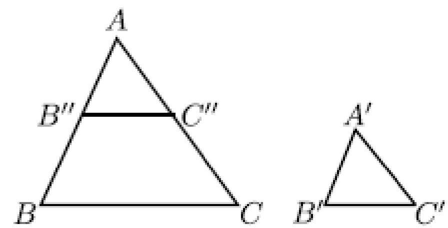
A line that is parallel to one side of a triangle cuts the other two sides in the same ratio.

The converse is also true: a line that cuts two sides of a triangle in the same ratio is parallel to the third side.

Theorem 13. (Textbook Page 315)
If two triangles are similar, then their sides are in proportion.
(Similar or equiangular triangles are separate triangles that have equal angles but may not have similar sides.)

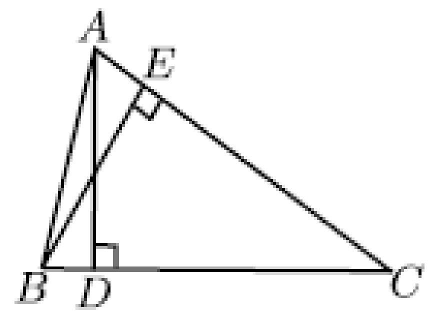
$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}.$$

The converse is also true: in any two triangles if matching sides are in proportion then the two triangles are similar to each other.



Theorem 16. (Textbook Page 316)

For a triangle, base times height does not depend on the choice of base.
(The area of a triangle is half the base by the height.)

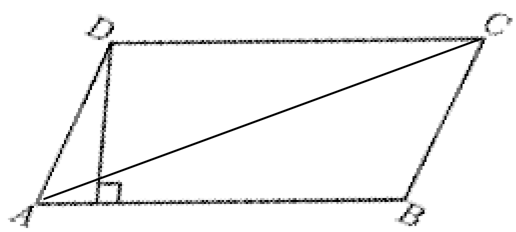


Congruent triangles have equal areas.

This is an example of a proposition whose converse is false. It may happen that two triangles have equal area, but are not congruent.

Theorem 17. (Textbook Page 310)

A diagonal of a parallelogram bisects the area.

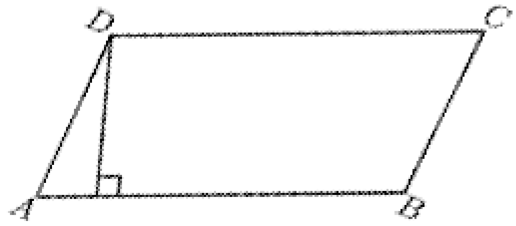


This theorem divides a parallelogram into two equal triangles of equal area, finding the area of one triangle and multiplying it by two gives the area of the parallelogram.

Theorem 18. (Textbook Page 310)

The area of a parallelogram is the base by the height.

The formula for the area of a parallelogram is in the log tables.

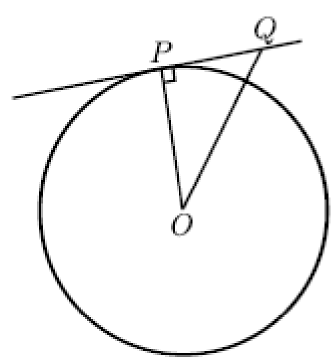


Circle Terms:

- A circle is a set of points that are all equidistant from a fixed point, its centre.
- A radius is a line from the centre of the circle to any point on the circle.
- A chord is a line from one side of a triangle to the other.
- A diameter is a chord that passes through the centre.
- The circumference is the perimeter or length of the circle.
- An arc is any part of the circumference.
- A sector is portion of a circle that is enclosed by the arc and two radii.
- A tangent is a line that touches the circle at only one point. The point where it touches the circle is the point of contact or the point of tangency.

Theorem 20. (Textbook Page 321)

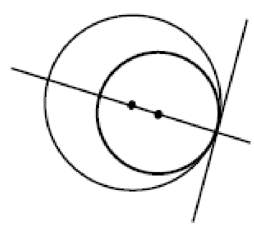
Each tangent is perpendicular to the radius that goes to the point of contact.
Converse: If the point P lies on the circle, and a line which passes through the point P is perpendicular to the radius at P, then the line is a tangent to the circle.

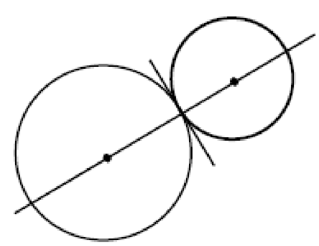


Corollary 6. (Textbook Page 322)
If two circles share a common tangent line at one point, then the two centres and that point are collinear.

(Collinear means that they sit on one line or are points on a single line.)

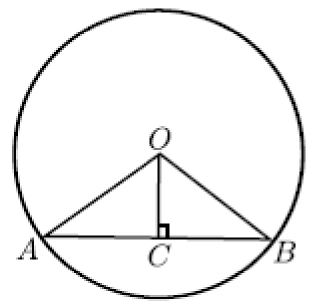
Two circles can meet internally or externally.





If two circles have just one point of intersection, then they are said to be touching and this point is referred to as their point of contact. The centres and the point of contact are collinear, and the circles have a common tangent at that point.

Theorem 21. (Textbook Page 320)



The perpendicular from the centre to a chord bisects the chord.
Converse: The perpendicular bisector of a chord passes through the centre.

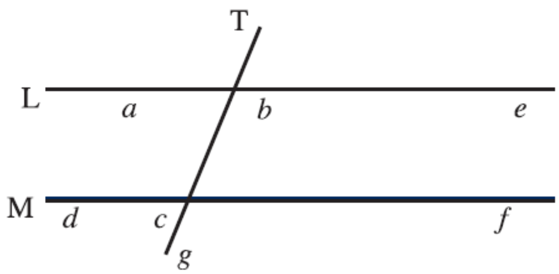
If a line is drawn at right angles to a chord and this line goes through the centre of the circle, it will cut the chord in two equal segments.

By constructing the perpendicular bisector of two chords you can locate the centre of a circle. The perpendicular bisectors will meet at the centre.

Brief summary of Junior Cert Ordinary Level Geometry (Old course)

You will not be asked to prove any of the theorems below but the question based on this, Paper 2 6B, is often very difficult, to date much more difficult than Q6A, so you should probably not answer it.

- *Construction*: To bisect an angle without using a protractor.
- *Construction*: To construct a triangle, given sufficient data.
- *Construction*: To construct the perpendicular bisector of a line segment without using a protractor or set square.
- *Construction*: To divide a line segment into three equal parts.
- A straight angle measures 180°.
- *Theorem*: Vertically opposite angles are equal in measure.
- Alternate angles are equal in measure when formed by two parallel lines intersecting a third line.
- Corresponding angles are equal in measure when formed by two parallel lines intersecting a third line.
- In the diagram below: (a) If $\angle abc = \angle bcf$ then line L is parallel to line M (b) If $\angle abc = \angle dcg$ then line L is parallel to line M



- *Theorem*: The measures of the three angles of a triangle sum to 180°.

- *Theorem*: An exterior angle of a triangle equals the sum of the two interior opposite angles in measure.

- Meaning of congruent triangles. Two triangles are congruent if they satisfy any one of the following four conditions:
  - three sides in one equal in measure to three sides in the other (SSS);
  - two sides and the included angle in one equal in measure, respectively, to two sides and the included angle in the other (SAS);
  - two angles and a side in one equal in measure, respectively, to two angles and a corresponding side in the other (ASA);
  - a right angle, hypotenuse and a side equal in measure, respectively, in each (RHS).

- *Theorem*: If two sides of a triangle are equal in measure, then the angles opposite these sides are equal in measure.

- *Converse*: If a triangle has two angles equal in measure, then the sides opposite these angles are equal in measure (i.e. the triangle is isosceles).

- If in a triangle two sides are of unequal length, then the angles opposite these sides are unequal in measure and the larger angle is opposite the longer side.

- Any two sides of a triangle are together greater in measure than the third side.

- The area of any rectilinear figure is equal to the sum of the areas of any two non-overlapping rectilinear figures of which it is composed.

- The area of a rectangle = length X breadth

- *Theorem*: Opposite sides and opposite angles of a parallelogram are respectively equal in measure.

- *Theorem*: A diagonal bisects the area of a parallelogram.

- *Theorem*: The diagonals of a parallelogram bisect each other.

- *Theorem*: The area of a triangle = ½base X (corresponding) perpendicular height.

- *Theorem*: The area of a parallelogram = base X (corresponding) perpendicular height.

- *Theorem*: An angle subtended by a diameter at the circumference is a right angle.

- *Theorem*: The sum of opposite angles of a cyclic quadrilateral is 180°.

- *Theorem (Theorem of Pythagoras)*: In a right-angled triangle, the square of the length of the side opposite to the right angle is equal to the sum of the squares of the lengths of the other two sides.

- *Converse of the Theorem of Pythagoras*: If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle has a right angle and this is opposite the longest side.